

Non-critical strings and \mathcal{W} -algebras

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Singularities of $\text{Spin}(7)$ manifolds are considered in the worldsheet approach, and it is argued that the internal CFT describing a singular $\text{spin}(7)$ has an enhanced $\mathcal{SW}(\frac{3}{2}, \frac{3}{2}, 2)$ algebra tensored with $\mathcal{N} = 1$ linear dilaton CFT, much like singular Calabi-Yau CFTs have a $\mathcal{N} = 2$ linear dilaton tensored with a suitable $\mathcal{N} = 2$ SCFT. Upon adding fundamental strings, these vacua are related to AdS_3 vacua with $\mathcal{N} = 1$ supersymmetry, completing the worldsheet classification of AdS_3 vacua with NS flux.

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1. Introduction

It is well known that the starting point for the description of Calabi-Yau compactifications in the RNS approach involves $\mathcal{N} = 2$ SCFTs with a charge integrality condition that allows GSO projection to preserve space-time supersymmetry. The other compactifications that can be similarly treated in RNS formalism are the $\text{spin}(7)$ and G_2 holonomy spaces. In this paper, we consider $\text{spin}(7)$ holonomy manifolds. It has been shown [1] that the data describing a $\text{spin}(7)$ compactification is the $\mathcal{SW}(\frac{3}{2}, 2)$ algebra with $c = 12$. This allows us to define GSO projection and preserve two supercharges in the uncompactified directions of \mathbf{R}^2 . We will review the derivation of this enhanced superconformal algebra in section 2, following an approach that was used in the case of Calabi-Yau compactifications. In the case of Calabi-Yau spaces it is well known that the CFT describing the compactification becomes singular as certain Kähler and/ or complex structure parameters are varied, and the singular CFT that one typically gets is based on the $\mathcal{N} = 2$ linear dilaton CFT tensored with a suitable internal SCFT with $\mathcal{N} = (2, 2)$ supersymmetry (the details of this CFT depend on the nature of the singularity). A natural generalization is to try and determine the type of singularities that a $\text{spin}(7)$ manifold can have, and describe the physics near this singularity in terms of a Liouville type theory. The straightforward approach to this question seems complicated because not much seems to be known about the moduli spaces of $\text{spin}(7)$ holonomy manifolds, and even less is known about the type of singularities that occur at a finite distance in moduli space. One can however note that the singular CFT is well described by a non-critical string, and that non-critical strings can be viewed as critical strings if we include the Liouville mode, so we are in essence looking for non-critical strings that contain a Liouville type mode, and further contain a $\mathcal{SW}(\frac{3}{2}, 2)$ SCA. As we show section 3, this leads naturally to the class of non-critical strings containing a linear dilaton CFT and a $\mathcal{SW}(\frac{3}{2}, \frac{3}{2}, 2)$ SCA. Alternatively, one can start with these non-critical backgrounds, and add fundamental strings that span \mathbf{R}^2 . Upon including back-reaction, one expects to end up with an $AdS_3 \times \mathcal{N}$ background, where \mathcal{N} is a suitable $\mathcal{N} = (1, 1)$ SCFT that allows string theory to preserve two supercharges in AdS_3 . For weakly coupled sigma models, we will show using supergravity that these backgrounds can preserve $\mathcal{N} = 1$ space-time SUSY only if there exists on the worldsheet a $\text{spin}-\frac{3}{2}$ operator that is

holomorphic. The minimal $\mathcal{N} = 1$ SCA that includes this spin- $\frac{3}{2}$ operator is nothing but the $SW(\frac{3}{2}, \frac{3}{2}, 2)$ algebra. $\mathcal{N} = 1$ $SW(\frac{3}{2}, \frac{3}{2}, 2)$ algebras have been recently discussed in a paper by Noyvert [2], to which we refer the reader for details.

2. Compact Spin(7) manifolds

We will consider Type II superstrings propagating in a geometry of the form $\mathbf{R}^{1,1} \times \mathbf{X}_8$, where the internal manifold is taken to correspond to an $\mathcal{N} = 1$ SCFT on the world-sheet, and will work in the RNS formalism. Then, it is well known how to construct the space-time supersymmetry vertex. In other words, one would like to know how to write down the target space-time supercharge, in terms of the world-sheet fields, so that space-time supersymmetry can be made explicit in the RNS version. The vertex operator for the space-time supercurrent is, in the $(-1/2)$ picture given as:

$$\mathcal{V}_\alpha = e^{-\phi/2} S_\alpha \Sigma \quad (2.1)$$

where ϕ is the bosonized super-ghost, S_α is the spacetime spin field, while the field Σ is the spin field coming from the Ramond sector of the internal SCFT to which the compactification on \mathbf{X}_8 corresponds.

An explicit formula for the spacetime spin field S , is given by bosonization [3]. Using the known OPEs of the fields ϕ and S , and insisting on spacetime supersymmetry, one easily arrives at the following OPEs for the fields Σ .

$$\Sigma(z)\Sigma(0) \sim \frac{1}{z} + O(z) \quad (2.2)$$

In order for the space-time supersymmetry vertex \mathcal{V}_α to give rise to a supercharge, the vertex must be an operator of dimension 1, and this forces the dimension of Σ to be $1/2$. This, together with the OPE (2.2), forces the field Σ to be a free fermion type theory [4]

This means, the form of the OPE of the field Σ , can be written down as follows:

$$\Sigma(z)\Sigma(0) \sim z^{-1} 1 + z\mathcal{O}(0) + \dots \quad (2.3)$$

In (2.3), \mathcal{O} is the stress tensor of the free fermion theory, and is an operator of dimension 2, which we will associate with the dimension 2 operator that appears in the extended superconformal algebra of [1]. The OPE of Σ with the superconformal generator T_F , is determined by the BRST invariance of the gravitino vertex, and forces the OPE to have terms no more singular than $z^{-1/2}$. That is,

$$\Sigma(z)T_F(0) \sim z^{-1/2} \quad (2.4)$$

Using the above OPE and the identification of the spin field Σ with the theory of free majorana fermions, one arrives at the following OPE between \mathcal{O} and T_F :

$$\mathcal{O}(z)T_F(0) \sim z^{-2}A(0) + z^{-1}\mathcal{W}(0) + \dots \quad (2.5)$$

Dimensional analysis tells us that A has dimension 3/2, while \mathcal{W} has dimension 5/2. This is the same as the dimension 5/2 operator that appears in [1]. As for A , since it has dimension 3/2 we will identify it with the supercurrent T_F .

This identification can easily be checked by referring back to the free fermion picture, where an explicit expression can be written down for \mathcal{O} in terms of Σ , using which one finds the OPE (2.5) with the identification of A with T_F as we claimed. More generally, as noted in [5], a symmetry current of weight 3/2 can only correspond to the supercurrent, in a unitary conformal field theory². So, the OPE (2.5) can be re-written as follows:

$$\mathcal{O}(z)T_F(0) \sim \frac{3}{4c} \frac{T_F(0)}{z^2} + z^{-1}\mathcal{W}(0) + \dots \quad (2.6)$$

² A way to see this is to note that, if the spin $\frac{3}{2}$ current is not the super-current, its OPE with the super-current forces the introduction of a super-partner, either of spin(1) or of spin($\frac{5}{2}$). The spin(1) current together with the rest of the symmetry currents forms the $\mathcal{N} = 2$ SCA (Calabi-Yau situation), whereas, with spin(2), the only way to close the OPE is with $c = \frac{21}{2}$ (this is the G_2 case), or we can append the G_2 algebra with a free $c = 1$ theory, which is nothing but a Spin(7) CFT again, as shown in [6] (geometrically this can be thought of as Spin(7)s which are S^1 fibered over a G_2 holonomy manifold).

In order to confirm the existence of this algebra, the OPEs between T , T_F , \mathcal{O} and \mathcal{W} , must close without introduction of further operators. The algebra discussed above closes only if $c = 12$, and the rest of the OPEs can be identified by the requirement of closure of the algebra [†]. Indeed the $\mathcal{SW}(\frac{3}{2}, 2)$ algebra is the unique $\mathcal{N} = 1$ SCA containing precisely one spin-2 multiplet in addition to the stress tensor and supercurrent of a generic $\mathcal{N} = 1$ SCA. Furthermore for $c = 12$ the $\mathcal{SW}(\frac{3}{2}, 2)$ algebra contains an Ising subsector [6] which is what forced $c = 12$ in our case.

In the above equations, c refers to the central charge of the internal $\mathcal{N} = 1$ super-algebra, which is determined from the requirement of the theory providing a good string background, to be $c = 12$, which is the same requirement for the closure of this algebra. To be more precise, the operator Σ was our spin-field, which created the Ramond ground state in the internal $\mathcal{N} = 1$ SCFT on \mathcal{N} , and this being the case, $\Delta[\Sigma] = \frac{c}{24}$ and hence the central charge of the internal SCFT is required to be 12. As noted, this is also the same value of the central charge, for which the extended algebra discussed above closes. Most of the proof in the above section assumed that the extended super-conformal algebra that arises from the requirement of $\mathcal{N} = 1$ supersymmetry in the target space, was minimal. It is in fact a generic situation for $\text{spin}(7)$ manifolds, though we will argue later that for certain singular $\text{spin}(7)$ manifolds there are additional conserved currents on the worldsheet.

Suppose we have $\mathcal{N} = 2$ supersymmetry in the external space-time, one should recover the extended super-conformal algebra underlying Calabi-Yau compactifications. It is easy to see that this is indeed the case. With twice as much supersymmetry in the target space, we have two spin-fields Σ_1 and Σ_2 , which satisfy the equations:

$$\Sigma_I(z)\Sigma_J(0) \sim \delta_{IJ}\frac{1}{z} + \dots \quad (2.7)$$

This means, we have two free majorana fermions, so we can use the usual bosonization prescription, and conclude that the theory possesses a $\text{spin}(1)$ conserved current, call it J . Then, the existence of this $\text{spin}(1)$ current means, as usual that the $\mathcal{N} = 1$ SCA on the world-sheet is enhanced to a $\mathcal{N} = 2$ SCA. The spin-field can be written in terms of the

[†] This algebra has been studied previously, in [6] and was first derived in [7]

bosonized R-current as $\Sigma_1 + i\Sigma_2 = e^{i\phi}$, where $J = i\sqrt{4}\partial\phi$.

Actually, the extended superconformal algebra associated with Calabi-Yau manifolds is in fact larger and contains as a subset the $\mathcal{N} = 2$ SCA. This larger algebra is generated by including the spectral flow generator and its $\mathcal{N} = 2$ superpartners. It can be easily checked that a \mathbf{Z}_2 projection of this larger algebra leaves a closed algebra of the form $\mathcal{SW}(\frac{3}{2}, 2)$ and is simply the statement that there are Spin(7) spaces that are obtained by orbifolding Calabi-Yau four-folds.

3. Singularities

It is familiar in the context of Calabi-Yau spaces, that the worldsheet theory becomes singular as certain Kähler and complex-structure parameters are tuned. At these points in moduli space, the worldsheet CFT acquires a “throat” and the physics down the throat is strongly coupled and describable in many cases as a Liouville type SCFT. The worldsheet CFT being strongly coupled coincides with certain non-perturbative objects like D-branes wrapping supersymmetric cycles becoming light. A very concrete prescription for the non-critical string theory describing a class of Calabi-Yau singularities was given in [8]. The worldsheet description of non-critical $\mathcal{N} = 1$ strings requires a linear dilaton multiplet tensored with a suitable SCFT \mathcal{N} . The linear dilaton multiplet consists of a scalar with background charge ϕ together with a Majorana fermion ψ_ϕ . In order to describe a string propagating on a spin(7) manifold we expect the internal CFT (the linear dilaton together with \mathcal{N} should provide a $\mathcal{SW}(\frac{3}{2}, 2)$ SCA. We shall argue that de-coupling the linear dilaton multiplet from this enhanced SCA gives rise to a $\mathcal{SW}(\frac{3}{2}, \frac{3}{2}, 2)$ algebra. In other words, a non-critical string that preserves two supercharges, can be constructed by tensoring the $\mathcal{N} = 1$ linear dilaton CFT with a matter CFT with the enhanced SCA $\mathcal{SW}(\frac{3}{2}, \frac{3}{2}, 2)$, with $c = \frac{21}{2} - 3Q^2$, where Q is the background charge of the linear dilaton.

The $\mathcal{N} = 1$ $\mathcal{SW}(\frac{3}{2}, \frac{3}{2}, 2)$ algebra is generated by the stress tensor T , the supercurrent G , a spin- $\frac{3}{2}$ super multiplet and a spin-2 super multiplet. This algebra exists for all c and a parameter λ called the self-coupling. For $c = \frac{21}{2}$ this is the G_2 algebra studied in [1]. For more details about the representation theory and the commutation relations that define the algebra we refer the reader to [2].

The straightforward way to show the claim is simply to start with a free-field representation of a $\mathcal{SW}(\frac{3}{2}, 2)$ algebra extended by a dimension $\frac{1}{2}$ multiplet (ψ_ϕ, ϕ) where ϕ carries background charge and show that one can de-couple this multiplet resulting in a $\mathcal{SW}(\frac{3}{2}, \frac{3}{2}, 2)$ SCA at $c = \frac{21}{2} - 3Q^2$.

In fact a calculation very similar to this has already been done in [6] where the authors showed that a $\mathcal{SW}(\frac{3}{2}, 2)$ algebra at $c = 12$ containing a $h = \frac{1}{2}$ multiplet could be re-written after de-coupling the free fermion and free current as the $c = \frac{3}{2}$ free theory tensored with a $\mathcal{SW}(\frac{3}{2}, \frac{3}{2}, 2)$ SCA at $c = \frac{21}{2}$. The only difference in our case is that the $h = \frac{1}{2}$ multiplet is not free. With minor changes their proof still goes through, showing that the tensor product of a $\mathcal{N} = 1$ linear dilaton together with $\mathcal{SW}(\frac{3}{2}, \frac{3}{2}, 2)$ algebra generates a spin(7) algebra which is extended.

We will however provide alternate arguments which we believe are more intuitive. We start with the observation that a large class of singular Calabi-Yau manifolds can be orbifolded to give rise to singular spin(7) geometries. We know that the non-critical string describing the singular Calabi-Yau geometries is a (deformation of) the $\mathcal{N} = 2$ linear dilaton CFT tensored with an internal $\mathcal{N} = 2$ SCFT. The orbifolding involves $J \rightarrow -J$. Our claim then amounts to the statement that the \mathbf{Z}_2 orbifold of the tensor product of S^1 with an $\mathcal{N} = 2$ SCA gives rise to a $\mathcal{SW}(\frac{3}{2}, \frac{3}{2}, 2)$ SCA.

One can view this as the CFT generalization of the familiar statement that a \mathbf{Z}_2 orbifold of a circle fibration of a Calabi-Yau three-fold gives rise to a G_2 holonomy manifold.

Since our analysis is going to be similar to the more familiar G_2 case, we will recall how the argument goes in the simpler setting. We know that a class of G_2 holonomy spaces can be obtained by considering $\frac{S^1 \times CY_3}{\mathbf{Z}_2}$. The worldsheet description of CY_3 compactifications is in terms of a $\mathcal{N} = 2$ SCFT with $c = 9$ and integral R-charges. As usual, let us bosonize the U(1) R-current by $J = i\sqrt{\frac{3}{2}}\partial\phi$. The two $\mathcal{N} = 2$ super currents are denoted by G^\pm , and the stress tensor by T . The S^1 part is simple enough, and given by a free fermion ψ_θ together with $\partial\theta$. The \mathbf{Z}_2 orbifold acts by $\psi_\theta \rightarrow -\psi_\theta$, $\theta \rightarrow -\theta$ and $\phi \rightarrow -\phi$. Tensoring the two CFTs together, is it possible to identify a $\mathcal{SW}(\frac{3}{2}, \frac{3}{2}, 2)$ algebra that is left invariant under the projection? The candidate for the spin- $\frac{3}{2}$ generator is:

$$H = e^{i\sqrt{3}\phi} + e^{-i\sqrt{3}\phi} + :J\psi_\theta: \quad (3.1)$$

Taking OPEs with the super-current $G = G^+ + G^- + \psi_\theta \partial \theta$, yields the spin-2 partner. The other generators are obtained from the HH and HM OPEs. The full set of generators form the G_2 superconformal algebra as they should.

Analogously singular CY 4-folds are obtained by tensoring the $\mathcal{N} = 2$ linear dilaton CFT (at $c = 3 + 3Q^2$) together with a $\mathcal{N} = 2$ SCFT at $c = 9 - 3Q^2$. GSO projection allows the presence of the following operator:

$$U =: e^{iQ\theta} e^{i\sqrt{\frac{2}{3}}\phi} : + e^{-iQ\theta} e^{-i\sqrt{\frac{2}{3}}\phi} : \quad (3.2)$$

which has spin $\frac{3}{2}$ and survives GSO projection. So the most general spin $\frac{3}{2}$ operator is of the form $H = U + : J\psi_\theta : + G$. Inclusion of this operator is expected to lead to the $\mathcal{SW}(\frac{3}{2}, \frac{3}{2}, 2)$ algebra at $c = \frac{21}{2} - 3Q^2$ upon orbifolding the $U(1)$ R-current by $J \rightarrow -J$ to go from CY 4-folds to spin(7)s. This orbifolding is the worldsheet description of the anti-holomorphic involution that gives us a spin(7) starting from a Calabi-Yau four-fold.

The $\mathcal{SW}(\frac{3}{2}, \frac{3}{2}, 2)$ algebra is determined in terms of two parameters the central charge c and the coupling λ . In fact the algebras of relevance to singular spin(7) manifolds must have λ fixed. Indeed it was shown by Noyvert that there is a real $\mathcal{N} = 1$ SCA with $c = \frac{7}{10}$ for any value of central charge $c \geq \frac{3}{2}$ if λ obeys:

$$\lambda = \frac{4(63 - 6c)(6c - 9)^2}{243(30c - 21)} \quad (3.3)$$

Of course, this real $\mathcal{N} = 1$ SCA is that of the tri-critical Ising model. The existence of this tri-critical Ising algebra allows us to decompose the conformal blocks of the internal $\mathcal{N} = 1$ SCFT such that we can preserve supersymmetry².

Now λ vanishes for $c = \frac{21}{2}$ where we recover the G_2 SCA. It also vanishes for $c = \frac{3}{2}$. This puts $Q = \sqrt{3}$. The $\mathcal{N} = 1$ SCFT with this enhanced SCA is nothing but the \mathbf{Z}_2 orbifold of the free boson on a circle of radius $\sqrt{3}$ together with a free boson. The orbifold acts by $\theta \rightarrow -\theta$ and $\psi_\theta \rightarrow -\psi_\theta$. Precisely this model arises upon decoupling the Liouville

² Indeed the fact that the presence of a Tri-critical Ising sector allows one to write a supersymmetric partition function was independently noticed by Eguchi et.al [9]. Our paper can be thought of as providing the conditions under which a Tri-critical Ising algebra exists in a spin(7) compactification.

field from $\mathcal{N} = 2$ super-Liouville theory with $c = 12$. In fact it is GSO projection that fixes $Q = \sqrt{3}$ in this case.

Let us consider the $\mathcal{N} = 2$ super-Liouville theory with $c = 12$. Asymptotically the theory is formulated in terms of a free $\mathcal{N} = 1$ SCFT with a compact boson θ and free Majorana fermion ψ_θ together with the $\mathcal{N} = 1$ linear dilaton system comprising ϕ (the Liouville field) and its superpartner ψ . This background is a singular Calabi-Yau 4-fold CFT if $c = 12$. Indeed the corresponding non-compact CY is nothing but the affine variety:

$$z_1^2 + z_2^2 + z_3^2 + z_4^2 + z_5^2 = 0 \in \mathbf{C}^5 \quad (3.4)$$

The deformation of this variety to make it non-singular corresponds to turning on a constant in the RHS of (3.4). It is well known that the effect of turning on μ is the same as turning on the worldsheet cosmological constant of the $\mathcal{N} = 2$ super-Liouville theory to which the non-compact CY (3.4) is mirror dual. The worldsheet cosmological constant is:

$$e^{-\varphi} e^{\frac{i}{Q}\theta} e^{\frac{1}{Q}\phi} \quad (3.5)$$

written in -1 picture. For $c = 12$, $Q = \sqrt{3}$ which implies θ lives on a circle of radius $\sqrt{3}$. Upon quotienting by the \mathbf{Z}_2 anti-holomorphic involution $\theta \rightarrow -\theta$, $\psi_\theta \rightarrow -\psi_\theta$ the free $c = \frac{3}{2}$ CFT of θ and ψ_θ becomes precisely the $\mathcal{N} = 1$ $\mathcal{SW}(\frac{3}{2}, \frac{3}{2}, 2)$ SCFT at $c = \frac{3}{2}$.

One of the examples treated in [9] was a coset based on $SO(7)/G_2$ where it was argued to correspond to a noncompact $\text{spin}(7)$ SCFT. Using the structure constants f_{abc} of the octonions, it is easy to see that there exist holomorphic spin- $\frac{3}{2}$ current $U = f_{abc}\psi^a\psi^b\psi^c$ and a spin-2 current $V = *f_{abcd}\psi^a\psi^b\psi^c\psi^d$ which are conserved, where ψ^a are the seven free fermionic superpartners of the coset. Together with the $\mathcal{N} = 1$ generators the currents (U, V) generate the $\mathcal{SW}(\frac{3}{2}, \frac{3}{2}, 2)$ algebra.

The $\mathcal{N} = 2$ supersymmetric worldsheet cosmological constant is invariant under the \mathbf{Z}_2 projection and gives us a $\mathcal{N} = 1$ superpotential which is invariant under the $\text{spin}(7)$ algebra. Interestingly enough, this is not the only deformation that one can do. In fact there exists a $\mathcal{N} = 1$ supersymmetric deformation that also preserves the $\text{spin}(7)$ SCA but does not arise by \mathbf{Z}_2 projection of a $\mathcal{N} = 2$ potential. This corresponds in the geometric terms to deforming away from the limit where the $\text{spin}(7)$ has $\pi_1 = \mathbf{Z}_2$ into a noncompact

spin(7) with trivial fundamental group. The relationship between this SCFT and the corresponding noncompact spin(7) manifold will be discussed elsewhere.

Even though we have motivated the existence of the G_2 like enhanced SCA, it would be nice to get a simpler explanation as to why there is such an extended algebra for singular spin(7)s. In the next section we show that vacua of linear dilaton (or AdS_3) type that have two supercharges also end up giving rise to conserved worldsheet currents in the standard way. The basic observation here is that starting with a singular spin(7) of the form $\mathbf{R}_\phi \times \mathcal{N}$ and adding F-strings takes us to a background of the form $AdS_3 \times \mathcal{N}$. One can then use supergravity to constrain the type of \mathcal{N} that gives rise to supersymmetric vacua. This constraint on the geometry of \mathcal{N} can be re phrased as the existence of certain holomorphic currents in the CFT describing string propagation on \mathcal{N} .

4. AdS_3 vacua that preserve two supercharges

Starting from the linear dilaton vacua discussed previously, one can construct vacua of AdS_3 type, by adding fundamental strings to vacua of the form $\mathbf{R}^{1,1} \times \mathbf{R}_\phi \times \mathcal{N}$. Considering a large number p of F1-strings spanning $\mathbf{R}^{1,1}$, we end up with $AdS_3 \times \mathcal{N}$. On the worldsheet, AdS_3 is described by a $\mathbf{SL}(2, \mathbf{R})$ Kac-Moody algebra at level k , so that taking k large the sigma model describing both AdS_3 and \mathcal{N} becomes weakly coupled and we expect supergravity to be a good approximation. In the supergravity limit, we shall examine the conditions for solutions of the form $AdS_3 \times \mathcal{N}$ to be supersymmetric vacua, and determine constraints on \mathcal{N} that arise this way. As we show below, the supergravity analysis directly reveals the presence of a holomorphic spin- $\frac{3}{2}$ and spin-2 operator, which together with their $\mathcal{N} = 1$ super partners give us the field content of a $\mathcal{SW}(\frac{3}{2}, \frac{3}{2}, 2)$ algebra.

Our starting point is an ansatz for the supergravity background of the form:

$$ds^2 = (AdS_3) + g_{mn} dx^m dx^n \quad (4.1)$$

In (4.1) g_{mn} refers to the metric on \mathcal{N} .

In order for part supersymmetry to be preserved in this background, the gravitino and dilatino variations must vanish. We are looking for backgrounds that are created by

NS5-branes and F-strings only, so the only fields that are turned on are the metric and 3-form NS-NS flux H . In order for the gravitino variation to vanish, we must have ⁴:

$$\nabla_M^\pm \epsilon^\pm = \nabla_M - \frac{1}{8} H_{MNP} \Gamma^{NP} \epsilon^\pm = 0 \quad (4.2)$$

For the dilatino variation to vanish, we need:

$$H^{MNP} \Gamma_{MNP} \epsilon^\pm = 0 \quad (4.3)$$

In (4.2) and (4.3) the indices M, N, P refer to the 10 dimensional space-time while lower case indices m, n, p refer to coordinates on \mathcal{N} and greek indices μ, ν parameterize AdS_3 . The dilaton is assumed to be constant for our background so we have not shown its dependence explicitly.

Respecting the $3 \oplus 7$ split of the metric, the ten dimensional spinors ϵ^\pm can be split as:

$$\epsilon^\pm = \eta \otimes \xi^\pm \quad (4.4)$$

The spinor η is a conformal Killing spinor of AdS_3 , so it satisfies the following equation:

$$\nabla_\mu \eta = \frac{k}{2} \gamma_\mu \eta \quad (4.5)$$

The constant k in (4.5) is related to the cosmological constant of AdS_3 . Since \mathcal{N} is a seven dimensional Euclidean manifold, the spinors ξ^\pm are Majorana. This means that in order to have two supercharges in AdS_3 (actually four including the superconformal generators), there must be precisely two nowhere vanishing spinor ξ^\pm in the internal space \mathcal{N} .

The ansatz for the H-flux consistent with maximal symmetry of AdS_3 is:

$$H_{\mu\nu\rho} = f \epsilon_{\mu\nu\rho} \quad (4.6)$$

The mixed components of H have to vanish, while H_{mnp} is arbitrary, subject to conditions following from (4.2) and (4.3). In (4.6), f measures the NS5-brane charge of the background.

⁴ The spinors ϵ^\pm are 10d Majorana-Weyl spinors of the same (opposite) chirality for type IIB(A)

In order for (4.3) to hold, we require:

$$\mathbb{H}\xi^\pm = \pm 6f\xi^\pm \quad (4.7)$$

where we used the contraction $\mathbb{H} = H_{mnp}\gamma^{mnp}$. This, along with (4.2) tells us that the spinor ξ must be covariantly constant with respect to a connection with torsion:

$$\nabla_m \xi^\pm \mp 12e^{-\frac{1}{2}\phi} H_{mnp} \gamma^{np} \xi^\pm = 0 \quad (4.8)$$

which can be written as:

$$\nabla_\pm \xi^\pm = 0 \quad (4.9)$$

The existence of a spinor ξ^+ satisfying (4.9) guarantees the existence of the three form $\Phi_{mnp} = \xi^{+T} \gamma_{mnp} \xi^+$, and the four-form $*\Phi$ that are also H^+ -covariantly constant.

The worldsheet description of this string background is a $\mathcal{N} = (1, 1)$ supersymmetric non-linear sigma model, whose action is:

$$S = \int d^2z (G_{ij}(\phi) + B_{ij}(\phi)) \partial\phi^i \bar{\partial}\phi^j + \psi_i (\bar{\partial}\psi^i + \Gamma_+{}^i{}_{jk} \bar{\partial}\phi^j \psi^k) \quad (4.10)$$

where B is the NS-NS 2-form field whose field strength is H . ϕ^i are coordinates on \mathcal{N} and ψ^i are the fermionic superpartners. As the background is a classical solution of string theory, the full action is order by order conformally invariant. However, the fields ψ^i and ϕ^i no longer obey the free-field equations. All OPEs acquire corrections that depend on the curvature of \mathcal{N} and depend on the parameter f . For $f \rightarrow 0$ we must recover the OPEs of the G_2 conformal algebra as in [1].

By using the classical equations of motion for the ψ field, the worldsheet operators $U = \Phi_{mnp} \psi^m \psi^n \psi^p$ and $X = *\Phi_{mnpq} \psi^m \psi^n \psi^p \psi^q$ are seen to be holomorphic. From their definition, it is apparent that U has spin- $\frac{3}{2}$ and V has spin-2⁵. Together with their $\mathcal{N} = 1$ superpartners, they have the right field content to form a $\mathcal{SW}(\frac{3}{2}, \frac{3}{2}, 2)$ algebra. Similarly there is an anti-holomorphic sector of conserved higher spin currents also (the theory is parity invariant upon $B \rightarrow -B$).

⁵ This is true even though ψ^i do not have canonical OPEs, as the spin of the fields are independent of k .

It has in fact been noticed by Giveon and Rocek [10] from the worldsheet point of view, that orbifolding by the $U(1)$ R-current of the internal $\mathcal{N} = 2$ SCA of a string vacuum of the form $AdS_3 \times S^1 \times \mathcal{M}$ gives rise to an $AdS_3 \times \mathcal{N}$ vacuum with $\mathcal{N} = 1$ supersymmetry. In that paper, it was also speculated that the internal SCFT corresponding to \mathcal{N} might have an enhanced SCA. The result of this paper are in complete agreement with the expectations of [10].

Furthermore, if we now have one more pair of Majorana spinors solving (4.7) and (4.9) it is easy to show that they define a nowhere vanishing vector field such that one can write $\mathcal{N} = S^1 \times \mathcal{M}$ with \mathcal{M} a Kähler manifold with torsion such that there are two-forms $J = \xi^{+T}_1 \gamma^{ab} \xi^+_2$ and $\bar{J} = \xi^{-T}_1 \gamma^{ab} \xi^-_2$ which are covariantly constant with respect to H^+ and H^- connections. This automatically leads to the worldsheet theory being described by a $U(1)$ fibration of a $\mathcal{N} = 2$ SCFT, which was found independently from the worldsheet perspective in [10].

5. Discussion

In this note we have pointed out that $\mathcal{N} = 1$ linear dilaton CFT can be tensored with a $\mathcal{SW}(\frac{3}{2}, \frac{3}{2}, 2)$ SCFT and GSO projected to form a consistent background which preserves two space-time supercharges. It is natural to expect that these vacua are related to singular limits of spin(7) CFTs, and indeed their construction shows the appearance of the enhanced SCA of spin(7)s naturally. It would be very interesting to obtain more examples of $\mathcal{SW}(\frac{3}{2}, \frac{3}{2}, 2)$ SCFTs in light of this fact. Moreover, there is a canonical way to resolve these singularities in the worldsheet theory by turning on a potential that is invariant under the spin(7) algebra. This would describe string propagating on certain noncompact spin(7) manifolds. It is quite non-trivial to identify the precise relationship between a given CFT and the geometric model it describes. In the $\mathcal{N} = 2$ situation remarkable simplicity arises because of the well known connection[11][12] between Landau-Ginzburg models and Calabi-Yau spaces realized as algebraic varieties in projective/ weighted projective spaces (which are thought of as defining the superpotential data of a $\mathcal{N} = 2$ supersymmetric UV free theory which flows to the corresponding Calabi-Yau sigma model in the IR). An analogous simplification does not seem to exist for spin(7) spaces. It would be very interesting

to determine the analog of LG/ CY correspondence for $\text{spin}(7)$ spaces. A typical $\mathcal{N} = 1$ supersymmetric LG theory does not have protected operators, in particular the superpotential does not parameterize the IR SCFT to which it conjecturally flows. Perhaps in the $\text{spin}(7)$ case the existence of the enhanced SCA might persist in some form even in the UV, which allows one to make non-renormalization type statements.

By adding F1-strings we were able to show that the AdS_3 vacua which preserve $\mathcal{N} = 1$ supersymmetry all have an enhanced SCA on the worldsheet of the form $\mathcal{SW}(\frac{3}{2}, \frac{3}{2}, 2)$. This completes the classification of AdS_3 vacua with NS-flux[13].

An entirely analogous discussion can be carried out to construct singular G_2 SCFTs. These SCFTs would not suffer from the 1-loop destabilization [14] that affects compactifications on eight dimensional manifolds and forces turning on RR-flux when the tadpole for B -field does not vanish in type IIA. Indeed the tadpole condition vanishes identically for seven dimensional manifolds allowing us to consider compactifications without RR fluxes.

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